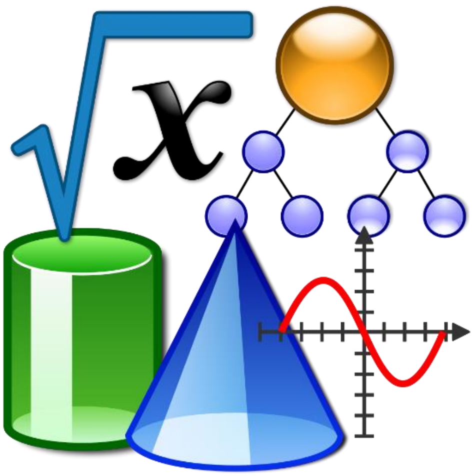
NPS Learning in Place

Algebra II



Name: ____________________School: _________________ Teacher:_____________

April 27 – May 15

Week 1	**Factoring**
Week 2	**Equations and Systems of Equations**
Week 3	**Sequences and Series**

Week 1

Factoring

During the school year, you have learned many methods for factoring such as, Greatest Common Factor (GCF), Difference of Two Squares (Difference of Perfect Squares), Difference of Cubes, and factoring a quadratic using either the box method, guess and check, or 'slip and slide', depending on your teacher. You've also learned how to use the x-intercepts of a polynomial to find its factors.

A polynomial of degree 4 is called a quartic. We will discuss, then practice, a method for factoring a quartic.

Factor: $2\mathbf{x^4 + 20x^2 + 48}$

We will start by treating the standard form of the polynomial as the "answer" and then figure out "what our original problem was" **-** the factored form. When finished, will have

(GCF) (factor)(factor)

1. **The first step is to ALWAYS factor out the GCF** (if it exists). If the leading coefficient is a negative, your GCF should also be negative. Since 2 is the greatest common factor we have

$$2(x^4 + 10x^2 + 24)$$

2. Next we want to factor

Put first (F) in first (F), last (L) in last (L) (**M is for middle**)

$$\mathbf{x^4 \quad + \quad 10x^2 \quad + \quad 24}$$

F M L

? ?

F **x^4**	M
M	**L** **24**

3) Multiply the First term the Last

$$(x^4)(24) = \mathbf{24x^4}$$

We also know that when we add the 2 M's (middles) our **SUM** must be equal to the middle term from our standard form, which was

$$\mathbf{10x^2}$$

Now, look for 2 monomials

whose product is $24x^4$ **<u>and</u>** whose sum is $10x^2$

We get

$6x^2$ and $4x^2$ (notice we have x^2 here and not x)

<table>
<tr>
<td>4. Now we have the other 2 terms that we need. It doesn't matter which middle box you put each in.</td>
<td>(GCF)2 ? ?
<table>
<tr><td>F
x^4</td><td>M
$\mathbf{6x^2}$</td></tr>
<tr><td>M
$\mathbf{4x^2}$</td><td>L
24</td></tr>
</table>
?

?</td>
</tr>
<tr>
<td>5. Find the greatest common factor of the first row (x^2 is the greatest common factor of x^4 and $6x^2$)</td>
<td>(GCF)2 ? ?
<table>
<tr><td>F
$\mathbf{x^4}$</td><td>M
$\mathbf{6x^2}$</td></tr>
<tr><td>M
$4x^2$</td><td>L
24</td></tr>
</table>
X^2

?</td>
</tr>
<tr>
<td>6. Now ask yourself what times x^2 give you the "answer of x^4 (box F). That of course would be x^2</td>
<td>(GCF)2 x^2 ?
<table>
<tr><td>F
$\mathbf{x^4}$</td><td>M
$6x^2$</td></tr>
<tr><td>M
$4x^2$</td><td>L
24</td></tr>
</table>
X^2

?</td>
</tr>
<tr>
<td>7. Now do the same thing with the other top box. Answer the question "x^2 times what will give you $6x^2$". Of course the answer is 6
Fill the answer in on the top.</td>
<td>(GCF)2 x^2 +6
<table>
<tr><td>F
x^4</td><td>M
$\mathbf{6x^2}$</td></tr>
<tr><td>M
$4x^2$</td><td>L
24</td></tr>
</table>
X^2

?</td>
</tr>
<tr>
<td>8. Now we need to fill in the last spot. Ask yourself the question "x^2 times what will give me $4x^2$?" The answer is 4. Fill in the last space. Check your last box. Is 6(4) = 24?

$2x^4 + 20x^2 + 48 = 2(x^2 + 6)(x^2 + 4)$</td>
<td>(GCF)2 x^2 + 6
<table>
<tr><td>F
x^4</td><td>M
$6x^2$</td></tr>
<tr><td>M
$\mathbf{4x^2}$</td><td>L
24</td></tr>
</table>
X^2

4</td>
</tr>
</table>

Factoring using grouping (4 terms)

Factor: $2x^3 + 3x^2 + 4x + 6$

(we will start with the "answer" and figure out what our original problem was**)**

F M M L

1. Since we already have 4 terms we will just be able to go directly to the box

2. **Put first (F) in first (F), last (L) in last (L) and put in the 2 Middle in M**

$2x^3$ + 6 $3x^2$ and 4x (order doesn't matter)

? ?

F $2x^3$	M $3x^2$
M $4x$	L 6

<table>
<tr>
<td>3. Find the greatest common factor of the first row
(x^2 is the greatest common factor of $2x^3$ and $3x^2$)</td>
<td>? ?
<table>
<tr><td>F
$2x^3$</td><td>M
$3x^2$</td></tr>
<tr><td>M
$4x$</td><td>L
6</td></tr>
</table>
X^2

?</td>
</tr>
<tr>
<td>4. Now ask yourself what times x^2 give you the "answer of $2x^3$ (box F).

That of course would be $2x$</td>
<td>2 x
<table>
<tr><td>F
$2x^3$</td><td>M
$3x^2$</td></tr>
<tr><td>M
$4x$</td><td>L
6</td></tr>
</table>
x^2
?</td>
</tr>
<tr>
<td>5. Now ask yourself what times x^2 gives you the "answer of $3x^2$ (top box M).
That of course would be 3.

Now ask yourself what time $2x$ will give you $4x$ (the bottom M).
It would be 2.

$2x^3 + 3x^2 + 4x + 6$ **= $(2x + 3)(x^2 + 2)$**</td>
<td>2x + 3
<table>
<tr><td>F
$2x^3$</td><td>M
$3x^2$</td></tr>
<tr><td>M
$4x$</td><td>L
6</td></tr>
</table>
X^2

+2</td>
</tr>
</table>

Factoring "difference of perfect squares"

Difference of squares of perfect squares polynomials can be, in general, be factored as such

$$a^2 - b^2 = (a + b)(a - b)$$

Ex. Factor $x^2 - 4$

Compare $x^2 - 4$

$a^2 - b^2 = (a + b)(a - b)$

therefore

$x^2 - 4 = (\quad + \quad)(\quad - \quad)$

$x^2 - 4 = (x+2)(x-2)$

Ex. Factor $64x^2 - y^2$

Compare $64x^2 - y^2$

$a^2 - b^2 = (a + b)(a - b)$

therefore

$64x^2 - y^2 = (\quad + \quad)(\quad - \quad)$

$64x^2 - y^2 = (8x+y)(8x-y)$

Ex. Factor $250m^2 - 10$

**GCF! GCF is 10. So, after factoring out the GCF, we have* $10(25m^2 - 1)$

Now compare $25m^2 - 1$

$a^2 - b^2 = (a + b)(a - b)$

therefore

$25m^2 - 1 = (\quad + \quad)(\quad - \quad)$

$25m^2 - 1 = (5m+1)(5m - 1)$ *but don't forget your GCF!*

$$\mathbf{10(5m+1)(5m - 1)}$$

1. **Factor completely using the box quartic method** $x^4 + 2x^2 – 24$	2. **Factor completely using the box- 4 terms** $x^3 + 2x^2 + 8x +16$
3. **Factor completely using difference of squares** $36x^2 - 25$	4. **Factor completely** $x^4 - 3x^2 – 54$
5. **Factor completely** $\mathrm{m}^4 - 4\mathrm{m}^2 – 54$	6. **Factor completely** $7y^4 + 43y^2 + 6$
7. **Factor completely** $b^5 + 13b^3 + 36$	8. **Factor completely** $x^4 - 3x^2 – 5$

9. **Factor completely**
$k^6 - 4k^4 – 5k^2$

10. **Factor completely**
$64x^4 - 1$

11. **Factor completely** $25x^2 - 9$	12. **Factor completely** $x^3 + 3x^2 - 4x$
13. **Factor completely** $x^4 + x^2 - 20$	14. **Factor completely** $x^3 - 2x^2 - 9x + 18$
15. **Factor completely** $45 - 80x^2$	16. **Factor completely** $2x^3 - 2x^2 - 5x + 5$
17. **Factor completely** $625 - 16m^4$	18. **Factor completely** $81x^2 - 16$
19. **Factor completely** $x^4 + 4x^2 - 21$	
20. **Factor completely** $1000x^2 - 490y^2$	

Week 2

Equations and Inequalities

AII.3 The student will solve
a) absolute value linear equations and inequalities;
b) quadratic equations over the set of complex numbers;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.

AII.4 The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically.

Absolute value linear equations and inequalities.

The absolute value is a number's **distance from zero.** Distance can only be positive so:

Ex: $|5| = 5$ Ex: $|-5| = 5$

When solving an absolute value equation such as $|x| = 5$ it's asking "what number(s) can I take the absolute value of in order to get 5?". As we can see above, there are 2! (note: if you ever see something like this, $|x| = -5$, there is no solution since absolute value is always positive.

Ex: $|x| = 10$ (split it into 2) $x = 10$ and $x = -10$. Notice that the x stays the same but the number on the other side splits in two equations where the number on the other side changes signs.

Process:

1. Isolate the absolute value expression. $|\text{expression}| = \textit{number}$
2. Determine the type of number the absolute value expression is equal to
   a. If it is equal to a NEGATIVE NUMBER the answer is NO SOLUTION.
   b. If it is equal to a POSITIVE NUMBER you will split into 2 equations (without absolute value bars)—(2 solutions)
   $\text{expression} = \textit{number}$ or $\text{expression} = -(\textit{number})$
   c. If it is equal to ZERO rewrite the equation without absolute value bars and solve for the variable. (one solution)

Ex1: $|x - 100| = 20$

Solution:

$$x - 100 = 20 \qquad\qquad x - 100 = -20$$

$$x - 100 + 100 = 20 + 100 \qquad\qquad x - 100 + 100 = -20 + 100$$

$$x = 120 \qquad\qquad x = 80$$

Ex2: $2|x-5|+1=7$ Solution:

$2|x-5|+1=7$ Original equation

$2|x-5|=6$ Subtract 1 from both sides

$|x-5|=3$ Divide by 2

$x-5=3$ $\quad x-5=-3$ Split into two equations

$x=8$ $\quad x=2$ Add 5 to both sides.

Ex3: $|3x-2|+8=1$ Solution:

$|3x-2|+8=1$ Original equation

$|3x-2|+8-8=1-8$ Subtract 8 from each side.

$|3x-2|=-7$ Simplify.

This sentence is *never* true. The solution set is $\varnothing$.

Ex3: $|x+10|=4x-8$ Solution:

$x+10=4x-8$

$10=3x-8$

$18=3x$

$6=x$

$x+10=-(4x-8)$

$x+10=-4x+8$

$5x+10=8$

$5x=-2$

$x=-\frac{2}{5}$

Assignment

1: $|2x+5|=13$

2: $|3x-4|-3=11$

3: $2|2x-5|+5=11$

4: $|2x+1|+4=4$

5: $2|2x-5|+9=9$

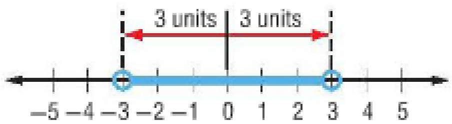
6: $4|x-1|+7=3$

7: $5-|2x-3|=7$

Absolute Value Inequalities and Solving Quadratics with Square Roots

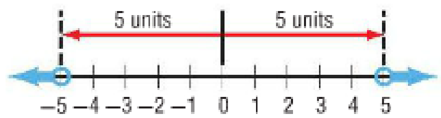
Absolute Value Inequalities:

Recall that absolute value means “distance from zero”. For instance $|x| < 3$ means that “where on the number line are numbers smaller than 3 units away from zero which looks like this:



Notice that the graph of $|x| < 3$ is the same as the graph of $x > -3$ and $x < 3$.

Therefore $|x| > 5$ would look like this:



Notice that the graph of $|x| > 5$ is the same as the graph of $x < -5$ or $x > 5$.

You can solve and graph these in almost the same way that we did with absolute value equations. However, when you change the sign on the second equation you also have to flip the sign of that inequality. As long as the x is on the left side, the direction to shade is indicated by the inequality (arrow).

Ex. **Solve $|6y - 5| \geq 13$. Graph the solution set on a number line.**

$|6y - 5| \geq 13$ is equivalent to $6y - 5 \geq 13$ or $6y - 5 \leq -13$. Solve the inequality.

$6y - 5 \geq 13$	or	$6y - 5 \leq -13$	Rewrite the inequality.
$6y \geq 18$		$6y \leq -8$	Add 5 to each side.
$y \geq 3$		$y \leq -\frac{8}{6}$ or $-\frac{4}{3}$	Divide each side by 6.

(note: if this was less than the shading would be in between these two values)

Day 2 Assignment #1 (Do the odds only)

Solve each and graph the solution.

1. $|2m| \geq 8$
2. $|-5y| < 35$
3. $|b-4| > 6$
4. $|6r-3| < 21$
5. $|3x-12| \leq 6$
6. $|5n-8| + 6 > 2$
7. $|2x-5| + 2 \leq 13$
8. $|4-3x| \geq 10$
9. $3|2x-1| + 5 > 14$
10. $y \geq |x+3|$
11. $y + 4 < |x-2|$
12. $2y - 4 < |x+3|$

13.

Which of the following is the inequality of the graph below?

a. $|3-2x| \geq 3$

b. $|3-2x| > 3$

c. $|3-2x| \leq 3$

d. $|3-2x| < 3$

Solving Quadratics by Square Roots:

Steps:

Square Root Principle

If $x^2 = k$, then $x = \pm\sqrt{k}$

Recall: a. $\sqrt{-27}$

$$\sqrt{-27} = \sqrt{-1 \cdot 3^2 \cdot 3}$$
$$= \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{3}$$
$$= i \cdot 3 \cdot \sqrt{3} \text{ or } 3i\sqrt{3}$$

Step 1: Isolate the perfect square.
Step 2: Take the square root of each side. Don't forget the $\pm$.
Step 3: Simplify the radical.
Step 4: Get x by itself.

Ex:
$$x^2 - 50 = 0$$
$$x^2 = 50$$
$$\sqrt{x^2} = \pm\sqrt{50}$$
$$x = \pm\sqrt{50}$$
$$= \pm\sqrt{2 \cdot 25}$$
$$= \pm 5\sqrt{2}$$

Ex:
$$(x-1)^2 - 5 = 7$$
$$(x-1)^2 = 12$$
$$\sqrt{(x-1)^2} = \sqrt{12}$$
$$x - 1 = \pm\sqrt{4 \cdot 3}$$
$$x = 1 \pm 2\sqrt{3}$$

Ex:
$$(x-5)^2 - 100 = 0$$
$$(x-5)^2 = 100$$
$$\sqrt{(x-5)^2} = \pm\sqrt{100}$$
$$x - 5 = \pm 10$$
$$x = 5 \pm 10$$
$$x = 5 - 10 \text{ or } x = 5 + 10$$
$$x = -5 \text{ or } x = 15$$

Day 2 Assignment #2:

Example: Now it's your turn. Solve $5t^2 - 125 = 0$.

First you need to isolate the squared term:

Do you now have $t^2 = 25$? If not, first add 125 to each side and then divide both sides by 5.

Now take the square root of each side.

Did you obtain $t = \pm 5$? If you only got one solution, what can you do to correct this?

Try These:

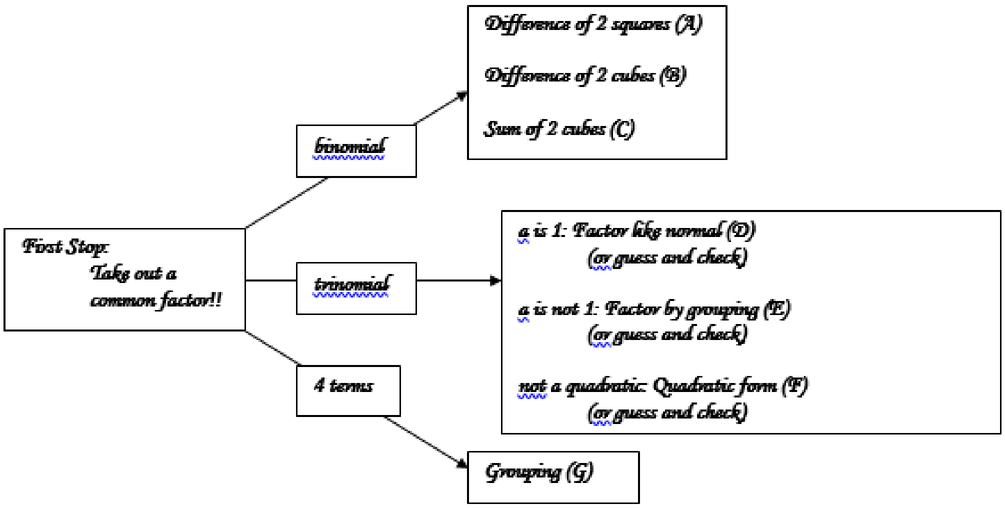
1). $4x^2 + 1 = 5$ (remember you should get 2 answers)

2). $3(x-1)^2 = 24$

3). $4x^2 + 13 = 13$ What do you notice about the number of solutions?

4). $3x^2 + 7 = 2x^2 - 5$ What do you notice about your answers? Be sure to simplify completely.

Factoring and Solving Polynomials



(A): Difference of 2 squares

$a^2 - b^2 = (a+b)(a-b)$

$ex:$

$4x^2 - 9 = (2x+3)(2x-3)$

$or:$

$x^6 - 25 = (x^3+5)(x^3-5)$

(B): Difference of 2 cubes

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$ex:$

$27x^3 - 64 = (3x-4)(9x^2+12x+16)$

(C): Sum of 2 cubes

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$ex:$

$8x^3 + 125 = (2x+5)(4x^2-10x+25)$

(D): a is 1: Factor like normal

ax^2+bx+c, but $a=1$

x^2+bx+c, find out what will multiply to get c and add to get b

$ex:$

$x^2 - 7x + 10 = (x-2)(x-5)$

(E): a is not 1: Factor by grouping

ax^2+bx+c, and $a \neq 1$

$ex:$

$3x^2 - 10x - 8$

$3x^2 - 12x + 2x - 8$

$3x(x-4) + 2(x-4)$

$(x-4)(3x+2)$

1. multiply a and c and find factors of that that will add to give you b.
2. split the middle term up using these two numbers.
3. pull out common factor of 1st two and last two.
4. factor out the set of parenthesis if they are the same

(F): not a quadratic: Quadratic form

If the highest exponent if bigger than 2, act like it is a quadratic.

ex:

$x^4 + 2x^2 - 24 = (x^2-4)(x^2+6)$

continue if possible... $= (x+2)(x-2)(x^2+6)$

(G): grouping

$2x^3 - x^2 - 6x + 3$

$x^2(2x-1) - 3(2x-1)$

$(2x-1)(x^2-3)$

Follow steps 3-4 on (E).

Assignment #1: Try to use the above road map in order to factor the following. Start on the left with "first step" and move in the direction that matches your polynomial. If you need help with that method find the example given below.

Quadratics

1). $3x^2 - 6$

2). $3x^2 - 12$ (make sure that you factor complely!

3). $x^2 - x - 12$

4). $2x^2 + 11x + 5$

5). $2x^2 + 8x + 6$ (remember to check the first step to factoring)

6). $10x^2 - 20x$

7). $x^3 + 1$

8). $27x^3 - 8$

9). $x^3 + 6x^2 - x - 6$

10). $x^4 - x^2 - 12$

11). $x^4 - 100$

Solving Polynomials by Factoring

Soooo….. why factor? One reason is that it became a completely different way to solve various equations.

KeyConcept Zero Product Property

Words	For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.
Example	If $(x + 3)(x - 5) = 0$, then $x + 3 = 0$ or $x - 5 = 0$.

Therefore, the solution to the equation above is $x = -3$ and $x = 5$. That works both ways. If I had a polynomial with solutions of $x = 7$ and say $x = \frac{1}{2}$ then the factors would be $(x - 7)(2x - 1)$.

Assignment #2

Write the factors of the polynomial given the following zeros:

1). $x = -1, x = 5$ 2). $x = 0, x = 1, x = -1$ 3). $x = \frac{2}{3}, x = \sqrt{2}, x = i$ challenge question

Solve the following using the zero product property:

4). $(x^2 - 6)(x^2 + 1) = 0$ 5). $(x - 2)(x + 2)(x^2 + 2) = 0$ 6). $x^3 - 4x = 0$

7). $x^3 + 5x^2 + 4x = 0$ 8). $x^4 - 2x^3 + 3x^2 - 6x = 0$ 9). $x^3 - 125 = 0$

Solving Radical and Rational Equations

Radicals

KeyConcept Solving Radical Equations

Step 1 Isolate the radical on one side of the equation.

Step 2 Raise each side of the equation to a power equal to the index of the radical to eliminate the radical.

Step 3 Solve the resulting polynomial equation. Check your results.

When solving radical equations, the result may be a number that does not satisfy the original equation. Such a number is called an **extraneous solution**.

Examples:

a. $\sqrt{x+2}+4=7$

$\sqrt{x+2}+4=7$	Original equation
$\sqrt{x+2}=3$	Subtract 4 from each side to isolate the radical.
$\left(\sqrt{x+2}\right)^2=3^2$	Square each side to eliminate the radical.
$x+2=9$	Find the squares.
$x=7$	Subtract 2 from each side.

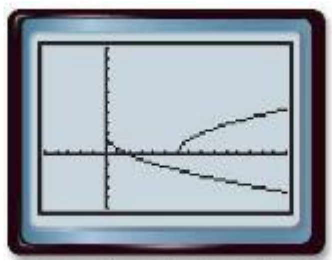
b. $\sqrt{x-12}=2-\sqrt{x}$

$\sqrt{x-12}=2-\sqrt{x}$	Original equation
$\left(\sqrt{x-12}\right)^2=(2-\sqrt{x})^2$	Square each side.
$x-12=4-4\sqrt{x}+x$	Find the squares.
$-16=-4\sqrt{x}$	Isolate the radical.
$4=\sqrt{x}$	Divide each side by -4.
$16=x$	Square each side.

c.

$2(6x-3)^{\frac{1}{3}}-4=0$	Original equation
$2(6x-3)^{\frac{1}{3}}=4$	Add 4 to each side.
$(6x-3)^{\frac{1}{3}}=2$	Divide each side by 2.
$\left[(6x-3)^{\frac{1}{3}}\right]^3=2^3$	Cube each side.
$6x-3=8$	Evaluate the cubes.
$6x=11$	Add 3 to each side.
$x=\frac{11}{6}$	Divide each side by 6.

Remember that you can find solutions using the calculator or DESMOS if available. This method can also tell you if the solution is extraneous! Just graph the left side and then the right.. If they don't cross, there's no solution.



Assignment #1:

Solve each equation.

1. $\sqrt{x-4}+6=10$

2. $\sqrt{x+13}-8=-2$

3. $8-\sqrt{x+12}=3$

4. $\sqrt{x-8}+5=7$

5 $\sqrt[3]{x-2}=3$

6. $(x-5)^{\frac{1}{3}}-4=-2$

Rationals

The easiest way to solve a rational equation is to use the least common denominator. You did this in the past using cross multiplication but you didn't know it.

Ex: Solve $\frac{2}{x} = \frac{4}{5}$ The least common denominator is $5x$ so we would multiply by that and get:

$$5x \cdot \frac{2}{x} = \frac{4}{5} \cdot 5x$$

The x's would divide out on the left and the 5's on the right leaving you with:

$$10 = 4x$$

$$\frac{5}{2} = x$$

Ex: **Example 1:** $\frac{2x}{3} + \frac{x-2}{5} = \frac{1}{6}$

Step 1: Multiply both sides of the equation by the least common multiple of the denominators. In this case, the LCM of 3, 5, and 6 is 30.

$$30 \times \left(\frac{2x}{3} + \frac{x-2}{5}\right) = \left(\frac{1}{6}\right) \times 30 \quad \Rightarrow 30\left(\frac{2x}{3}\right) + 30\left(\frac{x-2}{5}\right) = 5$$

Step 2: Simplify and solve familiar equation.

$\Rightarrow 20x + 6(x-2) = 5$

$\Rightarrow 26x - 12 = 5$

$\Rightarrow 26x = 17$

$\Rightarrow x = \frac{17}{26}$

Ex: **Example 2:** $\frac{4}{x+2} + \frac{5}{x-2} = \frac{29}{x^2-4}$

Step 1: Multiply both sides of the equation by the least common multiple of the denominators. In this case, the LCM of $x + 2$, $x - 2$, and $x^2 - 4$ is $(x + 2)(x - 2)$.

$$(x+2)(x-2)\left(\frac{4}{x+2} + \frac{5}{x-2}\right) = \left(\frac{29}{x^2-4}\right)(x+2)(x-2)$$

$$\Rightarrow (x+2)(x-2)\left(\frac{4}{x+2}\right) + (x+2)(x-2)\left(\frac{5}{x-2}\right) = \left(\frac{29}{x^2-4}\right)(x+2)(x-2)$$

Step 2: Simplify and solve familiar equation.

$$\Rightarrow \cancel{(x+2)}(x-2)\left(\frac{4}{\cancel{x+2}}\right) + (x+2)\cancel{(x-2)}\left(\frac{5}{\cancel{x-2}}\right) = \left(\frac{29}{\cancel{x^2-4}}\right)\cancel{(x+2)(x-2)}$$

$\Rightarrow 4x - 8 + 5x + 10 = 29$

$\Rightarrow 9x + 2 = 29$

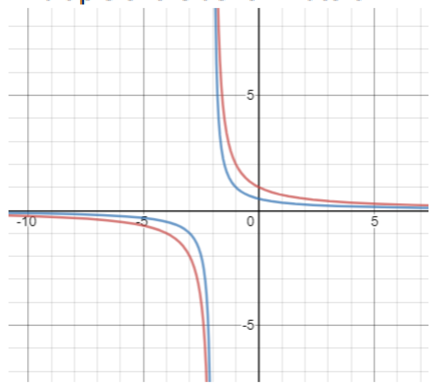
$\Rightarrow 9x = 27$

$\Rightarrow x = 3$

Ex: **Example 4:** $\frac{x+3}{x+2}=1-\frac{x+1}{x+2}$

Step 1: Multiply both sides of the equation by the least common multiple of the denominators. In this case the LCM is just $x + 2$.

$$\Rightarrow (x+2)\left(\frac{x+3}{x+2}\right)=\left(1-\frac{x+1}{x+2}\right)(x+2)$$



Step 2: Simplify and solve familiar equation.

$$\Rightarrow x+3=(x+2)-(x+1)$$
$$\Rightarrow x+3=1$$
$$\Rightarrow x=-2$$

Step 3: Verify solution.

When we check $x = -2$, we can see that it leads to division by zero. Because we correctly followed the process for finding a solution and the result we generated does not solve the equation, it is call an extraneous solution. Thus, the answer to the problem is “no solutions.”

Assignment #2**:**

Solve each of the following. Be sure to check solutions in the original equations and identify any extraneous solutions.
1. $\frac{4}{x}+\frac{1}{3x}=9$
2. $\frac{3}{n+1}=\frac{5}{n-3}$
3. $\frac{2}{x+5}-\frac{3}{x-4}=\frac{6}{x}$
4. $\frac{1}{x-5}+\frac{1}{x-5}=\frac{4}{x^2-25}$
5. $\frac{6x^2+5x-11}{3x+2}=\frac{2x-5}{5}$

Solving Linear-Quadratic and Quadratic-Quadratic Systems

If you remember from Algebra 1 you could solve a system of equations by either using substitution, elimination, or graphing. While graphing is the easiest method (with DESMOS you don't even have to solve for y first like you do for the TI's) we are going to focus on one method for non-linear systems. We are going to solve both equations for , set the equations equal to each other and then solve.

These can have a number of possible solutions as shown on the right:

Ex:

Solve the following system of equations: $y = x^2 - 6x + 9$ and $y + x = 5$.

Step 1 Solve $y + x = 5$ for y.

$y + x - x = 5 - x$ **Subtract x from both sides.**

$y = 5 - x$

Step 2 Write a single equation containing only one variable.

$y = x^2 - 6x + 9$

$5 - x = x^2 - 6x + 9$ **Substitute $5 - x$ for y.**

$5 - x - (5 - x) = x^2 - 6x + 9 - (5 - x)$ **Subtract $5 - x$ from both sides.**

$0 = x^2 - 5x + 4$

Step 3 Factor and solve for x.

$0 = (x - 4)(x - 1)$ **Factor.**

$x - 4 = 0$ or $x - 1 = 0$ **Zero-Product Property**

$x = 4$ or $x = 1$

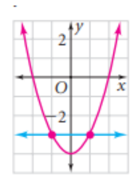
Step 4 Find the corresponding y-values. Use either equation.

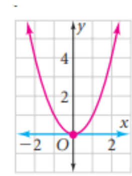
$y = -x^2 + 4x + 1$ $\quad\quad$ $y = -x^2 + 4x + 1$

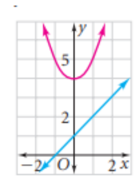
$= -(4^2) + 4(4) + 1$ $\quad\quad$ $= -(1^2) + 4(1) + 1$

$= 1$ $\quad\quad$ $= 4$

The solutions of the system are $(4, 1)$ and $(1, 4)$.


two solutions


one solution


no solutions

In order to see if an ordered pair is a solution to a system plug the ordered pair into both equations and make sure that you get a true statement for both

(meaning 4=4 etc…)

If you get even one false statement

(meaning 4=5) it is not a solution. This ordered pair must be where the graphs cross therefore it must be a solution for both.

Assignment:

Determine if the given ordered pair is a solution to the system of equations.

1). $(3,2)$; for $\begin{cases} y = 3x - 7 \\ y = (x-2)^2 + 1 \end{cases}$

2). $(3,2)$; for $\begin{cases} y = x^2 + 2x - 1 \\ y = -(x-3)^2 - 12 \end{cases}$

3). $(-1,-8)$; for $\begin{cases} y = x^2 + 3x - 6 \\ y = x^2 + 3x + 6 \end{cases}$

Solve each of the following using the method shown in the example above.

4). $\begin{cases} 2x + y = 3 \\ y = x^2 - 5x + 3 \end{cases}$

5). $\begin{cases} y = -3x + 5 \\ y = 2x^2 - 4x - 5 \end{cases}$

6). $\begin{cases} \boldsymbol{y = x^2 - 4} \\ \boldsymbol{y = 2x^2 + 5x} \end{cases}$

Week 3

SEQUENCES & SERIES

SOL AII.5: The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first *n* terms, determining the *n*th term, and evaluating summation formulas. Notation will include $\sum$ and a_n.

Lesson 1 Focus: Today we will investigate:

A. The general formula (or rule) for an arithmetic sequence

$$a_n = a_1 + (n - 1)d$$

where:

- a_n **is the n^{th} or general term**
- a_1 **is the first term**
- n **is the number of the term**
- d **is the common difference**

B. The use of the recursive formula

Use the sequence { 5, 8, 11, 14…} to answer questions 1 & 2.

1. Explain why this is an arithmetic sequence.
2. What is the common difference, d ?

---

3. Use the rule to find the first (5) terms of the sequence $a_n = 6 + (n - 1)(2)$

---

4. a) Use the general formula to find a_{14} if $a_1 = 12$ and $d = -4$
   b) Give the meaning of your answer

---

5. Write the general rule for the sequence {4, 1, −2, …}

---

6. Use the general formula to find a_{30} in the sequence $\left\{\frac{2}{3}, 1, \frac{4}{3}, \ldots\right\}$.

   Give the answer as a fraction.

---

7. Write the first five terms of the recursive sequence.

$$a_1 = -1$$
$$a_n = a_{n-1} - 5$$

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8. Write both the explicit (general) and recursive rules for the sequence

   {1, 8, 15, 22, 29, …}

Lesson 2 Focus: Today we will investigate the general formula (or rule) for a geometric sequence

$$\boldsymbol{a_n = a_1 r^{n-1}}$$

where:

a_n **is the** ***n*th or general term**
a_1 **is the first term**
r **is the common ratio**
n **is the number of the term**

Use the sequence {-2, -6, -18, …} to answer questions 1 & 2.

1. Explain why this is a geometric sequence.
2. What is the common ratio, *r* ?

3. Use the rule to find the first (4) terms of the sequence $a_n = 4(3)^{n-1}$

4. a) Use the general formula to find a_3 if $a_1 = 3$ and $r = 2$
   b) Give the meaning of your answer

5. a) Write the general rule for the sequence {3, 12, 48, …}
   b) Use your rule to find a_5

6. Which represents the explicit (general) rule, and which represents the recursive rule for the sequence {20, 10, 5, …} ? (two answers will not be used)

A $a_n = 20 + (n-1)(2)$

B $a_1 = 20$, $a_n = \frac{1}{2}(a_{n-1})$

C $a_n = 20(\frac{1}{2})^{n-1}$

D $a_1 = 20$, $a_n = 2(a_{n-1})$

7. Give the first (5) terms of the recursive sequence: $a_1 = 4$, $a_n = 2(a_{n-1})$

8. Write both the explicit (general) and recursive rules for the sequence

$$44, 11, \frac{11}{4}, \frac{11}{16}, \frac{11}{64}, \ldots$$

Lesson 3 Focus: Today we will investigate:

A. The general formula (or rule) for an arithmetic series

$$S_n = \tfrac{n}{2}\left[2a_1 + (n-1)d\right]$$

where:

S_n **is the sum of** n **number of terms**

a_1 **is the first term**

n **is the number of terms**

d **is the common difference**

B. The meaning and application of sigma notation, Σ

1. Explain the difference between the meaning of a_4 and S_4.

2. a) Use the formula to find S_8 if in a sequence if $a_1 = 6$ and $d = -5$

   b) Give the meaning of your answer

3. Use the appropriate formula to find S_{10} if the series is {-6 – 4.5 – 3 – 1.5 ...}

4. a) Give the meaning of: $\sum_{n=1}^{6} 3n - 4$

   b) Evaluate without using a formula

5. Write using sigma notation: 3 + 5 + 7 + 9 + 11 + 13

6. Given {6, 6.5, 7, 7.5, ...}

   a) Express S_n using sigma notation

   b) Use the appropriate formula to find S_6

Lesson 4 Focus: Today we will investigate:

A. The general formula (or rule) for a geometric series

$$S_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

where:

S_n **is the sum of** n **number of terms**

a_1 **is the first term**

n **is the number of terms**

r **is the common ratio**

B. The general formula for an infinite geometric series

$$S_\infty = \frac{a_1}{1 - r}$$

where:

S_∞ **is the sum of an infinite number of terms**

a_1 **is the first term**

r **is the common ratio**

1. Explain the difference between a sequence and a series.

2. a) Use the formula to find S_6 if in a sequence if $a_1 = 3$ and $r = 2$

   b) Give the meaning of your answer

3. a) Use the appropriate formula to find S_8 if the series is {4 – 8 + 16 – …}

4. Write using sigma notation: {4 + 8 + 16 + 32}

5. a) Give the meaning of $\sum_{n=1}^{5} 2(3^{n-1})$ b) Evaluate without using a formula:

6. Given {-2, -6, -18, …}

   a) Express S_n using sigma notation b) Use the appropriate formula to find S_4

7. a) Write the infinite series $\left\{\frac{2}{3} + \frac{1}{3} + \frac{1}{6} + \ldots\right\}$ using sigma notation

   b) Use the appropriate formula to evaluate the infinite series

Lesson 5 Focus: Today we will apply our understanding of arithmetic & geometric sequences & series. You will need the following formulas:

$$a_n = a_1 + (n-1)d \qquad a_n = a_1 r^{n-1}$$

$$S_n = \tfrac{n}{2}[2a_1 + (n-1)d] \qquad S_n = \frac{a_1(1-r^n)}{1-r} \qquad S_\infty = \frac{a_1}{1-r}$$

1. Given the sequence {4, 13, 22, 31, …}

   a) Write the explicit rule for the general term, a_n

   b) Write the recursive rule for the general term, a_n

2. Give the first (4) terms: $a_n = 3(2)^{n-1}$

3. Without using a formula, find $\sum_{x=1}^{4}(2x-1)$

4. Use the general formula to find the 14th term in the sequence {1, –1, –3, …}

5. Express the series using sigma notation: {–2 – 1 – ½ – ¼ }

6. Use the appropriate formula to find S_{18} if in a sequence if $a_1 = 8$ and $d = -5$

7. Use the appropriate formula to find the sum of the infinite series $\left\{-\frac{3}{2}+\frac{3}{4}-\frac{3}{8}\ldots\right\}$

8. Ms. Crusty holds a Multiplication Facts contest in her classroom. The winner is the one who can answer the most multiplication problems in a minute. To prepare, Carl answers 4 more questions per minute than the previous day, starting with 15 questions in one minute on the first day.

Use the appropriate formula to determine algebraically how many multiplication questions Carl will answer on the fifth day.

9. An auditorium has 25 seats in the first row, 29 seats in the second row, 33 seats in the third row, and so on for 36 rows. Use the appropriate formula to determine how many total seats there are in the auditorium.